Optimal Formation Networks of Multiple Dynamic Agents

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Abstract: In the paper, we consider the optimal communication networks for dynamic formation of multi-agent systems. Quadratic cost functions are established for both continuous-time and discrete-time multi-agent systems. For continuous-time multi-agent systems, we obtain that the optimal formation networks are star graphs. For discrete-time multi-agent systems, we prove that the optimal formation networks are still star graphs. Numerical simulations are presented to illustrate the effectiveness of the obtained theoretical results.

Key Words: Multi-agent systems, formation, linear-quadratic-regulator

1 Introduction

In recent years, more and more attention was attracted in the field of distributed cooperative control of multi-agent systems (MASs). This is primarily due to the fact that there are so many applications of MASs, to name but a few, cooperative control of unmanned air vehicles [1], formation of robots [2], distributed estimation over sensor networks [3, 4], and so on. Research interests in multi-agent coordination pertain to consensus [5, 6], containment control [7–9], flocking [10], formation control [2, 11], etc.

Among above aspects of MASs, consensus is a fundamental topic. Since it is the basement of many applications of MASs, there are tremendous literature in this field. To date, first-order consensus [12], second-order consensus [13], finite-time consensus [14], leader-following consensus [15, 16] of MASs have been widely investigated. Formation control of multi-agent system is one of the applications. Over the past decade, formations of unmanned air vehicles [2], autonomous surface vessels [11] and underwater robots [17] are studied by more and more researchers. Many researchers have investigated multi-agent formation problem under different agent dynamics and various tasks demanded. In 1990, [18] considered that a group of mobile robots form to circles and simple polygons. Lin et al. [19] gave the necessary and sufficient graphical conditions for formation control of a group of autonomous unicycles. Formation of nonholonomic mobile robots was also considered in [20]. The authors studied formation control in a leader-follower framework for nonholonomic mobile robots with input constraints. In practice, the dynamics of the agents might be different. Consequently, cooperative control for heterogeneous multi-agent systems is considered in [1] and [21]. Zheng et al. [1] propose a novel protocol to form a formation and to distinguish two different kinds of agents. Ji et al. [22] considered the optimal formation control of quasi-equilibrium for a group of agents with a leader.

Motivated by above results, this paper considers optimal communication network problem for formation control. By utilizing the linear-quadratic-regulator (LQR) theory, we prove that the optimal networks for dynamic formation control of second-order continuous-time agents are star graphs. For second-order discrete-time agents, the optimal formation networks are also star graphs. The results mean that, in order to minimize the cost function of formation, each follower should connect to the leader with different weights in the position and the velocity graph and do not communicate with other agents.

The rest part of this paper is organized as follows. Section 2 gives the preliminary knowledge about graph theory and LQR theory. Section 3 presents our main results. Simulations are provided in Section 4 to illustrate the effectiveness of the theoretical results. Finally, a short conclusion is given in Section 5.

Notations: let $\mathbb{R}$ be the set of real numbers. $\mathbb{R}^{n \times m}$ is the set of $n \times m$ real matrices. Denote $1_n$ (or $0_n$) as the column vector with all entries equal to one (or all zeros). Notation diag($a_1, \cdots, a_n$) represents the diagonal matrix with $a_i, i = 1, \ldots, n$, on its diagonal.

2 Preliminaries on Algebraic Graph Theory

2.1 Graph Theory

In this subsection, we introduce some basic notions about algebraic graph theory. For more details, please refer to [23].

Let $G = (V, E, A)$ be a weighted directed graph with the vertex set $V = \{1, 2, \ldots, n\}$, the edge set $E \subseteq V \times V$. An edge of $G$ is denoted by $(j, i)$, where $j$ is called the parent vertex of $i$. Nonnegative matrix $A = [a_{ij}]$ is weighted adjacency matrix of $G$, where $a_{ij} > 0$ if $(j, i) \in E$ and $a_{ij} = 0$ otherwise. The neighbor set of node $i$ denoted by $N_i = \{j \in V : (j, i) \in E \}$. A graph is called star graph if there exists a node $i_0$ satisfies: 1) it is the parent vertex of all other vertices; 2) for all $i \neq i_0$, $N_i = \{i_0\}$; 3) $N_0 = \emptyset$.

2.2 Infinite-Time Linear Quadratic Regulator Theory

Let

$$\dot{X}(t) = AX(t) + BU(t),$$

be a continuous-time linear system where $X(t) \in \mathbb{R}^n$, $U(t) \in \mathbb{R}^m$, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. Consider the optimal control problem

$$\min_{U(\cdot)} J(U(\cdot), X(0)) = \int_0^\infty \left[ X^T(t)QX(t) + U^T(t)RU(t) \right] dt$$

s. t. $\dot{X}(t) = AX(t) + BU(t)$

where symmetric matrices $Q \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$ are nonnegative and positive definite, respectively.
**Lemma 1** [24] Suppose that system (1) is completely controllable or completely stabilizable. Then, the optimal control problem (2) has a unique optimal control input \( U^*(t) = -R^{-1}B'PX(t) \) where \( P \) is the unique positive-definite solution of the algebraic Riccati equation (ARE)

\[
A^TP + PA + Q - PB^{-1}B^TP = 0.
\]

Let \( X(k + 1) = AX(k) + BU(k), k \in \mathbb{N} \)

be a discrete-time linear system where \( X(k) \in \mathbb{R}^n, U(k) \in \mathbb{R}^m \), \( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times m} \). Define the optimal control problem

\[
\min_{U(t)} J(U(.), X(0)) = \sum_{k=0}^{\infty} X^T(k)QX(k) + U^T(k)RU(k) \tag{4}
\]

s.t. \( X(k + 1) = AX(k) + BU(k) \),

where \( Q \in \mathbb{R}^{m \times m} \) and \( R \in \mathbb{R}^{m \times m} \) be symmetric, and be non-negative and positive definite, respectively.

**Lemma 2** [24] Suppose that system (3) is completely controllable or completely stabilizable. Then, the optimal control problem (4) has a unique optimal control input \( U^*(k) = -(R + B'B)^{-1}B'PAX(k)X(k) \) where \( P \) is the unique positive-definite solution of the discrete-time ARE

\[
A^TP - PB(R + B'B)^{-1}B^TPA + Q = P. \tag{9}
\]

### 3 Main Results

#### 3.1 Optimal formation networks of continuous-time multi-agent systems

Consider a multi-agent system which is composed of \( n \) second-order integrator agents. Let \( V = \{1, 2, ..., n\} \) be the set of agents. Each agent dynamics can be written as

\[
\begin{cases}
\dot{\xi}_i(t) = v_i(t), \\
\dot{v}_i(t) = u_i(t), 
\end{cases} \tag{5}
\]

where \( \xi_i \in \mathbb{R}^N, v_i \in \mathbb{R}^N \) and \( u_i \in \mathbb{R}^N \) are the position, velocity and control input, respectively, of agent \( i \). Since agents of (5) are of second-order, the position and the velocity information exchanging should be considered. Thus, we define \( G_p = (V, E_p, A) \) and \( G_v = (V, E_v, W) \) as the position communication network and the velocity communication network, respectively. We assume that agent \( n \) is a leader. Thus, the Laplacian matrix of \( G_p \) and \( G_v \) should be written as

\[
L_p = \begin{pmatrix} L^1_p & -\mathbf{b} \\ 0 & 0 \end{pmatrix},
L_v = \begin{pmatrix} L^1_v & -\mathbf{d} \\ 0 & 0 \end{pmatrix}.
\]

The control input of agent \( i \) is \( u_i(t) = 0 \).

**Definition 1** For a given vector \( h(t) = [h^1(t), h^2(t), ..., h^n(t)]^T \in \mathbb{R}^{nN} \), system (5) is said to achieve dynamic formation \( h(t) \) if for any given bounded initial states,

\[
\lim_{t \to \infty} (\xi_i(t) - h_i(t) - r(t)) = 0,
\lim_{t \to \infty} (v_i(t) - v(t)) = 0, i \in V
\]

hold, where \( r(t) \) is a formation position reference function and \( v(t) \) is a formation velocity reference function.

Throughout this paper, we suppose that \( h_i(t) \) is a constant vector. For agents \( 1, ..., n-1 \), we propose the following control inputs:

\[
u_i(t) = \sum_{j=1}^{n} a_{ij}(\xi_j(t) - h_j) - (\xi_i(t) - h_i) + \sum_{j=1}^{n} w_{ij}(v_j(t) - v_i(t)). \tag{7}\]

Therefore, the position and the velocity of agent \( i \) will converge to \( \xi_i(t) + h_i \) and \( v_i(t) \), respectively. Thus, \( \xi_i(t) \) and \( v_i(t) \) are the position reference function and the reference velocity function of the formation. We define error for agents \( i \):

\[
\begin{align}
\epsilon_i(t) &= \xi_i(t) - \xi_i(t) - h_i, \\
\zeta_i(t) &= v_i(t) - v_i(t), 
\end{align} \tag{8}
\]

Let \( e(t) = [\epsilon_1^T(t), ..., \epsilon_{n-1}^T(t), \zeta_1^T(t), ..., \zeta_{n-1}^T(t)]^T \) and \( u(t) = [u_1^T(t), ..., u_{n-1}^T(t)]^T \). Therefore, we define the formation cost by

\[
J(u(t), e(t)) = \int_0^\infty \left\{ \sum_{i=1}^{n-1} (\alpha_i \epsilon_i^T(t) \epsilon_i(t) + \beta_i \zeta_i^T(t) \zeta_i(t) + \lambda_i u_i^T(t) u_i(t)) \right\} dt. \tag{9}\]

In this paper, the objective is to design communication networks for system (5) to achieve a distributed dynamic formation with minimum cost. The optimal formation problem can be formulated as

\[
\min_{u(t)} J(u(t), e(0)) \tag{10}\]

s.t. (6) and (7).

It follows that the dynamics of error system can be written as

\[
\begin{cases}
\dot{e}(t) = \begin{pmatrix} 0 & I_{n-1} \\ 0 & 0 \end{pmatrix} \otimes I_N e(t) + \begin{pmatrix} 0 \\ I_{n-1} \end{pmatrix} \otimes I_N u(t), \\
\dot{u}(t) = -[L^1_p, L^1_v] \otimes I_N e. \tag{11}\end{cases}
\]

It is not hard to find that solving (10) is equivalent to designing the optimal communication networks \( G^*_p \) and \( G^*_v \).

**Theorem 1** For optimal formation problem (10), the optimal formation networks \( G^*_p \) and \( G^*_v \) are all star graphs. More specifically, each agent \( i \in \{1, 2, ..., n - 1\} \) is only connected to the leader \( n \) with the weight \( \sqrt{\alpha_i / \lambda_i} \) in \( G^*_p \) and the weight \( \sqrt{2 \alpha_i / \lambda_i + \beta_i / \lambda_i} \) in \( G^*_v \).

**proof.** It follows from (11) that the optimal formation problem (10) can be converted into the following optimal control problem

\[
\min_{u(t)} J(u(t), e(0)) \tag{12}\]

s.t. (11).

For system (11), the controllability matrix is

\[
\begin{pmatrix} 0 & I_{n-1} & \cdots \\ I_{n-1} & 0 & \cdots \end{pmatrix} \otimes I_N.
\]

Therefore, we know that the error system (11) is controllable. By lemma 1, we get that optimal control problem
have a unique optimal feedback control input \( u^*(t) = -(\Lambda^{-1} B^T) \otimes I_N Pe(t) \) where \( \Lambda = \text{diag}[\lambda_1, \ldots, \lambda_{n-1}] \). \( P \) is the unique positive-definite solution of the associated ARE

\[
P(G \otimes I_N) + (G \otimes I_N)^T P + Q \otimes I_N - \lambda P(B \otimes I_N)(\Lambda^{-1} \otimes I_N)(B^T \otimes I_N)P = 0,
\]

where \( G = \left( \begin{array}{cc} 0 & I_{n-1} \\ 0 & 0 \end{array} \right) \), \( B = \left( \begin{array}{c} 0 \\ I_{n-1} \end{array} \right) \) and \( Q = \text{diag}[\alpha_1, \ldots, \alpha_{n-1}, \beta_1, \ldots, \beta_{n-1}] \). Suppose that \( P = Y \otimes I_N \) where \( Y \in \mathbb{R}^{(n-1) \times (n-1)} \). From (13), we can obtain that \( Y \) is the unique positive-definite solution of the following ARE

\[
YG + G^T Y + Q - YBA^{-1}B^T Y = 0.
\]

We assume \( Y = \left( \begin{array}{ccc} Y_1 & Y_2 & Y_3 \end{array} \right) \). It follows that

\[
Y_2\Lambda^{-1}Y_2^T = \text{diag}[\alpha_1, \ldots, \alpha_{n-1}],
\]

\[
Y_2\Lambda^{-1}Y_3 = Y_1,
\]

\[
Y_3\Lambda^{-1}Y_3 = Y_3 + Y_2^T + \text{diag}[\beta_1, \ldots, \beta_{n-1}] = \sum_{i=1}^{n-1} Y_i + \text{diag}[\beta_1, \ldots, \beta_{n-1}].
\]

Thus, we get

\[
Y_1 = \text{diag}\left\{ \sqrt{\alpha_2(2\sqrt{\alpha_1}A_1 + \beta_1)}, \ldots, \sqrt{\alpha_{n-1}(2\sqrt{\alpha_{n-2}}A_{n-1} + \beta_{n-1})} \right\},
\]

\[
Y_2 = \text{diag}\left\{ \sqrt{\alpha_1A_1}, \ldots, \sqrt{\alpha_{n-1}A_{n-1}} \right\},
\]

\[
Y_3 = \text{diag}\left\{ \sqrt{A_1(2\sqrt{\alpha_1}A_1 + \beta_1)}, \ldots, \sqrt{A_{n-1}(2\sqrt{\alpha_{n-1}}A_{n-1} + \beta_{n-1})} \right\}.
\]

Therefore, we know that the optimal feedback control input is

\[
u^*(t) = -(\Lambda^{-1} B^T) \otimes I_N Pe(t) = -(\Lambda^{-1} Y_2, \Lambda^{-1} Y_3] \otimes I_N Pe(t).
\]

Since the feedback matrix is corresponding to the communication networks of multi-agent system (5). It is easy to know that the corresponding Laplacian matrices of optimal communication networks are

\[
L_p^* = \left( \begin{array}{cc} \text{diag}[a_1^*, \ldots, a_{n-1}^*] & -b \\ 0 & 0 \end{array} \right)
\]

and

\[
L_v^* = \left( \begin{array}{cc} \text{diag}[w_1^*, \ldots, w_{n-1}^*] & -d \\ 0 & 0 \end{array} \right),
\]

where \( a_i^* = \frac{\alpha_i}{\sqrt{\alpha_1}} \) and \( w_i^* = \frac{\sqrt{w_i}}{\sqrt{\alpha_1}} \). Therefore, \( L_p^* \) and \( L_v^* \) are Laplacian matrices of star graphs which means \( G_p^* \) and \( G_v^* \) are star graphs. If

3.2 Optimal formation networks of discrete-time multi-agent systems

In this subsection, we consider a multi-agent system consisting of \( n \) discrete-time second-order agents. Agent \( n \) is a leader and agents 1, 2, \ldots, \( n-1 \) are followers. The dynamics of every agents is

\[
\begin{cases}
\xi_i(k+1) = \xi_i(k) + v_i(k), \\
v_i(k+1) = v_i(k) + u_i(k), k = 1, 2, \ldots, n.
\end{cases}
\]

The control input of leader \( n \) is \( u_n(t) = 0 \). For second-order discrete-time multi-agent system (15), a dynamic formation can be defined as follows.

**Definition 2** For a given vector \( h = [h_1^T, h_2^T, \ldots, h_n^T]^T \in \mathbb{R}^{n^2} \), system (5) is said to achieve dynamic formation \( h(k) \) if for any given bounded initial states,

\[
\lim_{k \to \infty} (\xi_i(k) - h_i - r(k)) = 0,
\]

\[
\lim_{k \to \infty} (v_i(k) - v(k)) = 0, i = 1, 2, \ldots, n,
\]

hold, where \( r(k) \) is a formation position reference function and \( v(k) \) is a formation velocity reference function.

In order to achieve a formation, we propose the following control inputs:

\[
u_i(k) = \sum_{j=1}^{n} a_{ij}(\xi_j(k) - h_j) + w_{ij}v_j(k), i = 1, 2, \ldots, n-1.
\]

When multi-agent system (15) achieves formation \( h \), the position and the velocity of agent \( i \) will converge to \( \xi_i(k) + h_i \) and \( v_i(k) \) respectively. Therefore, we let \( e(k) = [e_1^T(k), e_2^T(k), \ldots, e_n^T(k), \xi_n^T(k)]^T \) where \( e_i(k) = \xi_i(k) - \xi_n(k) - h_i \) and \( \xi_n(k) = v_n(k) - w_nu_n(k) \). The cost function is defined by

\[
J(u(k), e(k)) = \sum_{k=0}^{\infty} \left( \sum_{i=1}^{n} \right) |e_i(k)|^2 + \beta\xi_n^2(k) + \lambda u_n^2(k)
\]

Then, we consider the following optimal formation problem

\[
\min_{u(k)} J(u(k), e(k))
\]

s. t. (16) and (17).

**Theorem 2** Optimal formation problem (19) will be solved when the position and the velocity communication networks \( G_p^* \) and \( G_v^* \) of system (15) satisfy

1) both two networks are star graphs and the leader \( n \) is the center;
2) the weights of \( G_p^* \) and \( G_v^* \) is decided by

\[
\begin{cases}
[a_1^*, w_1^*] = \left( \begin{array}{c} a_1 \\ 0 \end{array} \right),
\end{cases}
\]

\[
\begin{cases}
L_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\end{cases}
\]

where \( P_i \in \mathbb{R}^{2 \times 2} (i = 1, 2, \ldots, n-1) \) is the solution of the discrete-time ARE

\[
\begin{pmatrix}
0 & 1 \\
1 & 0 
\end{pmatrix} = P_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & \beta_1 \end{pmatrix} - P_1,
\]

\[
\begin{pmatrix}
0 & 1 \\
1 & 0 
\end{pmatrix} = P_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & \beta_1 \end{pmatrix} - P_1,
\]

\[
P_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 0.
\]
Proof. Let \( u(k) = [u_1^T(k), \ldots, u_{n-1}^T(k)]^T \). It is easy to know that the dynamics of the \( e(k) \) can be written as

\[
e(k+1) = I_{n-1} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_N e(k) + I_{n-1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \otimes I_N u(k),
\]

\[
u(k+1) = -[L_p^L \otimes (1 0) L_l^L \otimes (0 1)] \otimes I_N e(k).
\]

Therefore, we find that optimal formation problem (19) is an optimal control problem. Moreover, the discrete ARE of the problem is

\[
Q \otimes I_N + \left( I_{n-1} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_N \right) P \left( I_{n-1} \otimes \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \otimes I_N \right)
\]

\[- P - \left( I_{n-1} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_N \right) P \left( I_{n-1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \otimes I_N \right) \]

\[- \left[ \Lambda \otimes I_N + (I_{n-1} \otimes (0 1) \otimes I_N) \left( I_{n-1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \otimes I_N \right) \right]^{-1} (I_{n-1} \otimes (0 1) \otimes I_N) P \left( I_{n-1} \otimes \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \otimes I_N \right) = 0
\]

\[
(22)
\]

where \( Q = \text{diag}[^{\alpha_1, \beta_1, \ldots, \alpha_{n-1}, \beta_{n-1}}] \) and \( \Lambda = \text{diag}[\lambda_1, \ldots, \lambda_{n-1}] \). Easy to find that \( P = \text{diag}[P_1, P_2, \ldots, P_{n-1}] \otimes I_N \) where \( P_i \) is the solution of the following ARE

\[
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} P_i \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \alpha_i & 0 \\ 0 & \beta_i \end{pmatrix} - P_i - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} P_i \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left[ \lambda_i + (0 1) P_i \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \right]^{-1} (0 1) P_i \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = 0
\]

\[
(23)
\]

It should be mentioned that (23) is the ARE of the following optimal control problem

\[
\min_{u(k)} \sum_{k=0}^{\infty} \left[ \alpha_k \epsilon_k^T(k) \epsilon_k(k) + \beta_k \zeta_k^T(k) \zeta_k(k) + \lambda_k u_k^T(k) u_k(k) \right]
\]

s. t. \( \epsilon(k+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \epsilon_k(k+1) + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} u_k(k) \)

\[
(24)
\]

As a result, we know that \( u_k(k) = b^*_k \epsilon_k(k) + d^*_k \zeta_k(k) \) is the optimal control input, where

\[
[b^*_k, d^*_k] = \left[ \lambda_i + (0 1) P_i \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \right]^{-1} (0 1) P_i \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.
\]

Recalling (21), we have \( L_p^L = \text{diag}[b^*_1, \ldots, b^*_{n-1}] \) and \( L_l^L = \text{diag}[d^*_1, \ldots, d^*_{n-1}] \) which means the optimal communication networks are star graphs.

4 Simulations

Table 1: Cost function under three different pairs of networks

<table>
<thead>
<tr>
<th>((G^<em>_p, G^</em>_l))</th>
<th>((G^<em>_p, G^</em>_l))</th>
<th>((G^<em>_p, G^</em>_l))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.0493 \times 10^4)</td>
<td>(1.3013 \times 10^4)</td>
<td>(1.6730 \times 10^4)</td>
</tr>
</tbody>
</table>

In this section, suppose that four second-order agents will achieve a line formation. Assume that \( \alpha_i = \beta_i = \lambda_i = 1 \).

We consider the formation control under three pairs of communication networks \( (G^*_p, G^*_l), (G^*_p, G^*_l) \) and \( (G^*_p, G^*_l) \) which are shown in Fig. 1, Fig. 3 and Fig. 5, respectively. The formation procedure under these three pairs of networks are given in Fig. 2, Fig. 4 and Fig. 6. Table 1 presents the costs \( J(u(t), e(0)) \) under different communication networks with same initial states. It is shown that the cost under \( (G^*_p, G^*_l) \) is minimal. Comparing procedures in Fig. 2, Fig. 4 and Fig. 6, we find the formation is achieved more quickly under \( (G^*_p, G^*_l) \). Fig. 7 shows the position communication network \( G_p \) and the velocity network \( G_v(y, z) \) with the weights \( \sqrt{3}, \sqrt{3} \), \( y \in \mathbb{R}^+ \) and \( z \in \mathbb{R}^+ \). Thus, cost function is a function of \( y \) and \( z \). We present cost function with a random chosen initial states in Fig. 8. It is shown that cost function achieve minimum when \( y = z = \sqrt{3} \). Those results are consistent with the result of Theorem 1.

5 Conclusion

LQR-based optimal communication networks for dynamic formation have been investigated in this paper. Firstly, we proposed a quadratic cost function for dynamic formation of continuous-time multi-agent systems. It is shown that the optimal formation networks are star graphs which each follower just connects to the leader. Secondly, by LQR method, we can prove the optimal formation networks for dynamic formation of discrete-time multi-agent systems are star graphs. Future work may consider optimal formation control problems for some MASs with constrains, such as MASs with fixed or switching topologies, and MASs without velocity measurement.

References

Fig. 1: The optimal communication networks \((G_p^*, G_v^*)\)

Fig. 2: A formation under communication networks \((G_p^*, G_v^*)\) with cost \(1.0493 \times 10^4\)

Fig. 3: A pair of communication networks \((G_p^2, G_v^2)\)

Fig. 4: A formation under communication networks \((G_p^2, G_v^2)\) with cost \(1.3013 \times 10^5\)

Fig. 5: Communication networks \((G_p^3, G_v^3)\)

Fig. 6: A formation under communication networks \((G_p^3, G_v^3)\) with cost \(1.6730 \times 10^4\)

Fig. 7: The communication networks \((G_p, G_v(y, z))\)

Fig. 8: Cost function under communication networks \((G_p, G_v(y, z))\)


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