Robust Stabilization of Networked Control Systems with Multiple-packet Transmission via Jump System Approach

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Abstract: A jump system approach to stabilization and robust stabilization of networked control systems (NCSs) with multiple-packet transmissions are addressed. We focus our attention on the case that the packets are transmitted via limited capacity communication channels. Sufficient conditions on the mean square stabilization of NCSs are obtained in terms of linear matrix inequalities (LMIs). Non-fragile state feedback controller can be constructed directly via LMIs. A numerical example is worked out to demonstrate the effectiveness of the proposed method.

Key Words: Networked control systems, multiple-packet transmission, jump system, robust stabilization, LMIs

1 Introduction

In recent years, networked control systems (NCSs) has received increasing attention in control field [9], [15]. NCSs are feedback control systems with the control system components (sensors, controller, actuator, etc) connected via a real-time network. NCSs have many attractive advantages, such as reduce the cost of cable and power, ease of system diagnosis and maintenance, and increase the reliability. However, the insertion of communication network in the feedback control loop complicates the analysis and design of an NCS because many ideal assumptions made in the traditional control theory can not be applied to NCSs directly. It should be noted that there are some inevitable unfavorable effects, such as communication constraints [7]-[9], uncertainty caused by network transmission [6], multiple packets transmissions and packet dropouts [16, 20]. These problems may deteriorate the control performance or even destroy the stability of the system. So how to control such NCSs is a big problem.

Reference [19] models networked control system with multiple-packet transmission as a linear switching model with certain switching rules. Output feedback controller is constructed in terms of linear matrix inequalities (LMIs). In [21], multiple-packet transmission and short time delay is considered, where the packet transmission sequence is deterministic. But in the real networked control system with multiple-packet transmission, the packet transmission sequence is generally non-deterministic. A lot of work has been committed to a class of stochastic linear systems subject to variations governed by a Markov process. Reference [2] studies stochastic stability, stabilizability and and $H_\infty$ disturbance attenuation of discrete-time linear time-delay systems with Markovian jumping parameters using the stochastic Lyapunov function approach. However, multiple-packet transmission is not considered in this paper. [4] considers the case of multiple-packet transmission under dynamic scheduling strategy. It is modeled as a class of stochastic system, the stability of the system is analyzed by using the method of stochastic system. None of them has considered the effects of the uncertainties.

Uncertainty is common in control systems and it inevitably exists in system modeling due to the complexity of the system itself, exogenous disturbance, measurement errors and so on. Some results have studied the effects of the uncertainties for NCSs. [5] concerns the problem of $H_\infty$ output tracking for NCSs. Both network-induced delays and data packet dropouts have been taken into consideration. [12] discusses robust $H_\infty$ control problems for NCSs with time delays and subject to norm-bounded parameter uncertainties. In [17], robust $H_\infty$ control is considered for a class of networked systems with random communication packet losses and norm-bounded parameter uncertainties. An observer-based feedback controller is designed to robustly exponentially stabilize the system by solving certain LMIs. However, none of them has considered the effect of the uncertainties in the controller and the effect of limited communication. It is necessary to introduce some uncertain parameters in the modeling of NCSs, and design robust stabilizing controllers.

Motivated by the references above, this paper propose a jump system approach to stabilize NCSs with uncertainties and multiple-packet transmission. The main contribution of this paper is to develop non-fragile stabilizing controllers for such NCSs. Constant non-fragile stabilizing controllers are developed for the NCSs in terms of linear matrix inequalities, which can be easily solved by Matlab toolbox [1]. Moreover, the constant controllers are often effective and are easy to implement and maintain by plant personnel. For simplicity, we consider the sensors are clock-driven, the controller and the actuator are event-driven. The controller will use the old state measurement if there is no new data updating.

This paper is structured as follows. Section 2 models uncertain NCSs with multi-packet transmission as jump linear systems. Section 3 presents robust stabilization and non-fragile state feedback controller design results for state feedback case. Section 4 presents a numerical simulation to illustrate the efficiency of our approach. Section 5 concludes this paper.

Notation: We use standard notations throughout this essay. Denote $A^T$ the transpose of a matrix $A$. $A > 0$ ($A < 0$) means that $A$ is positive definite (negative definite). $I$ is the
identify matrix of appropriate dimension.

2 System Modeling

In distributed NCSs, due to the wide location of sensors whose information length may surpass the capacity of network, so multiple-packet transmission is necessary. We consider the case that all the nodes communicate via a limited bandwidth communication channel. State information of the system is split into different packets and only one packet can be transmitted at a time. The packet transmission process is governed by a Markov chain [10] with finite state-space.

An NCS with multiple-packet transmission can be described in Fig.1. The uncertain NCS is expressed by

$$x(k + 1) = (A + \Delta A)x(k) + Bu(k), \quad k = 1, 2, \ldots,$$

and the controlled input is

$$u(k) = (F + \Delta F)x(k) + \bar{F}(k),$$

where $$x(k) \in \mathbb{R}^n$$, $$u(k) \in \mathbb{R}^p$$ are the plant state and the plant input, respectively. $$A, B$$ are known real constant matrices with appropriate dimensions. $$F$$ is the feedback gain to be designed. $$\bar{F}(k)$$ is the content of the register, $$\Delta F$$ is variations of the controller gain and $$\Delta A$$ characterizes the uncertainty in the system, which satisfy the following assumption.

$$\Delta A = ET(k)H, \quad \Delta F = GT(1)J.$$  (3)

with $$E$$, $$H$$, $$G$$ and $$J$$ being known real constant matrix function with Lebesgue-measurable elements which satisfies $$\Gamma(k)^T \Gamma(k) \leq I, \Gamma(1)^T \Gamma(1) \leq I, \forall k$$. 

We assume that the state is split into d packets $$\bar{x}(k) = [X_1^T(k), \ldots, X_d^T(k)]$$, where $$X_i(k) = [x_{r_i+1}(k), \ldots, x_{r_i(k)}]^T$$. The controller will use the content of the register $$\bar{x}(k) = [X_1^T(k), \ldots, X_d^T(k)]$$, where

$$\bar{x}(k) = \begin{cases} X_i(k), & \text{if the packet containing } X_i(k) \text{ is transmitted;} \\ X_i(k - 1), & \text{otherwise.} \end{cases}$$

The packet transmission sequence of sensor nodes is non-deterministic under dynamic scheduling strategy. The states of markov chain expressed by $$\{S_k\}$$ are utilized to model the packet transmission process, which takes values in a finite set $$S = \{0, 1, \ldots, d\}$$, where

$$S_k = \begin{cases} i, & \text{if the } i^{th} \text{ packet's state information is transmitted;} \\ 0, & \text{otherwise} \end{cases}$$

The markov chain with a transition probability matrix $$P = [P_{ij}]$$ is given by

$$P_{ij} = \text{Prob}(S_{k+1} = j | S_k = i), P_{ij} \geq 0,$$

and the controller is

$$u(k) = (F + \Delta F)[\Lambda_{s_k} x(k) + (I - \Lambda_{s_k})\bar{x}(k - 1)].$$

Now, we can write the evolution of the closed-loop system as

$$x(k + 1) = ((A + \Delta A) + B(F + \Delta F)\Lambda_{s_k})x(k) + B(F + \Delta F)(I - \Lambda_{s_k})\bar{x}(k - 1).$$

Define

$$\Phi_{s_k} = \left[ A + \Delta A, B\Lambda_{s_k}, B\Lambda_{s_k}(I - \Lambda_{s_k}) \right].$$

3 Stability Analysis and Stabilization Result

The NCS (7) is a general jump linear system with the Markovian integer jump parameter $$\Phi_{s_k} \in \{\Phi_0, \Phi_1, \ldots, \Phi_d\}$$. Clearly, the original system (1)-(2) is stable if system (7) is stable. So we only need to prove that system (7) is stable.

First, we study the stabilization of the NCS (7) without uncertainties. We know the closed-loop system without uncertainties can be described as:

$$Z(k + 1) = \Psi_{s_k}Z(k),$$

where

$$\Psi_{s_k} = \left[ A + BF\Lambda_{s_k}, BF(I - \Lambda_{s_k}) \right].$$

Before proceeding, we need the following definition.

Definition 1. [11] NCS (1)-(2) is mean square (MS) asymptotically stable, if for all initial state $$(Z_0, S_0)$$, $$\lim_{k \to \infty} E[\|Z(k)\|^2] = 0$$ holds, where $$E$$ is the statistical expectation operator.
Below, we present necessary and sufficient matrix inequality conditions on MS stable of the system (8), which has been proved in [3].

**Lemma 1.** ([3]) Jump linear system (8) is said to be MS stable, if there exist symmetric positive definite matrices $Q_m, \ (m = 0, 1, \cdots, d)$ satisfying the following condition:

$$\Psi_m^T (\sum_{n=0}^{d} P_{mn}Q_n) \Psi_m < Q_m, \ \ m = 0, 1, \cdots, d \quad (9)$$

With a freedom matrix $S$ introduced, we will develop new sufficient conditions on the stabilization of the NCS. The following lemma is a different form of Theorem 1 in [13]. We present the following MS stabilization result on NCS (8).

**Lemma 2.** Jump linear system in (8) is said to be MS stable, if there exist symmetric positive definite matrices $Q_m, \ (m = 0, 1, \cdots, d)$, and matrices $K, Y, S$ satisfying $SB = BK$ and the following LMI:

$$
\begin{bmatrix}
\hat{Q}_m \\
\sqrt{p_{mn}W^T} \\
\sqrt{p_{mn}}W^T \\
\sqrt{p_{mn}W^T} \\
\sqrt{p_{mn}}W^T \\
\hat{S} + \hat{S}^T - \hat{Q}_0 \\
\sqrt{p_{mn}}W^T \\
\hat{S} + \hat{S}^T - \hat{Q}_1 \\
\sqrt{p_{mn}}W^T \\
\hat{S} + \hat{S}^T - \hat{Q}_d
\end{bmatrix}
> 0,
$$

then NCS (8) can be MS stabilized with the state feedback gain

$$F = K^{-1}Y, \quad (11)$$

where

$$W = \begin{bmatrix}
SA + BY\Lambda_s & BY(I - \Lambda_s) \\
S\Lambda_s & S(I - \Lambda_s)
\end{bmatrix},$$

$$\hat{Q}_m = \begin{bmatrix}
Q_m & 0 \\
0 & Q_m
\end{bmatrix}, \ \hat{S} = \begin{bmatrix}
S & 0 \\
0 & S
\end{bmatrix}. \quad (12)$$

The following lemma will play a key role to give a sufficient condition on the robust mean square stability of NCS (1).

**Lemma 3.** ([14]) Let $M, N, A$ be given matrices of compatible dimensions with $A$ satisfying $A^T A \leq I$, then the following inequality holds for any positive scalar $\epsilon > 0$

$$MAN + NT^T A^T M^T \leq \epsilon MMM^T + \epsilon^{-1}N^TN$$

In the following part we consider the robust mean square stabilization of NCS (1).

**Theorem 1.** NCS (1)-(2) is robust MS stable if there exist symmetric positive definite matrices $Q_m, (m = 0, 1, \cdots, d)$, positive scalars $\epsilon, \hat{\epsilon}$ and matrices $K, Y, S$ satisfying $SB = BK$ and

$$\begin{bmatrix}
-\Omega_1 & \tilde{M}^T & \tilde{N} & \tilde{L}^T & \tilde{V} \\
\tilde{M} & -\Omega_2 & 0 & 0 & 0 \\
\tilde{N}^T & 0 & -\Omega_3 & 0 & 0 \\
\tilde{L} & 0 & 0 & -\Omega_4 & 0 \\
\tilde{V}^T & 0 & 0 & 0 & -\Omega_5
\end{bmatrix}< 0, \quad (13)$$

then NCS can be MS stabilized with the state feedback gain

$$F = K^{-1}Y, \quad (14)$$

where

$$W = \begin{bmatrix}
SA + BY\Lambda_s & BY(I - \Lambda_s) \\
S\Lambda_s & S(I - \Lambda_s)
\end{bmatrix},$$

$$\begin{bmatrix}
\hat{Q}_m \\
\sqrt{p_{mn}W^T} \\
\sqrt{p_{mn}}W^T \\
\sqrt{p_{mn}W^T} \\
\sqrt{p_{mn}}W^T \\
\hat{S} + \hat{S}^T - \hat{Q}_0 \\
\sqrt{p_{mn}}W^T \\
\hat{S} + \hat{S}^T - \hat{Q}_1 \\
\sqrt{p_{mn}}W^T \\
\hat{S} + \hat{S}^T - \hat{Q}_d
\end{bmatrix},$$

$$\Omega_1 = \begin{bmatrix}
\epsilon I & 0 & \cdots & 0 \\
0 & \epsilon I & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \epsilon I
\end{bmatrix},$$

$$\begin{bmatrix}
\epsilon I & 0 & \cdots & 0 \\
0 & \epsilon I & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \epsilon I
\end{bmatrix},$$

$$\Omega_4 = \Omega_5 = \begin{bmatrix}
\epsilon M & 0 & \cdots & 0 \\
0 & \epsilon M & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \epsilon M
\end{bmatrix},$$

$$\begin{bmatrix}
\epsilon L & \epsilon L & \cdots & \epsilon L \\
0 & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0
\end{bmatrix},$$

$$L = \begin{bmatrix}
-(I\Lambda_s)^T & 0 \\
-(I(I - \Lambda_s))^T & 0
\end{bmatrix},$$

$$\tilde{N} = \tilde{N} = \begin{bmatrix}
0 & N_{m0} & \cdots & N_{md} \\
0 & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0
\end{bmatrix},$$

$$N_{mn} = \sqrt{p_{mn}}E^TS^T 0 \quad 0 \quad 0,$$

$$\begin{bmatrix}
0 & V_{m0} & \cdots & V_{md} \\
0 & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0
\end{bmatrix},$$

$$V_{mn} = \sqrt{p_{mn}(SBG)^T 0 \quad 0 \quad 0}, (n = 0, \cdots, d).$$
Proof. From lemma 2, system (7) is robust mean square stable if there exist symmetric positive definite matrices 
\( \hat{Q}_m(n = 0, 1, \cdots, d) \) satisfy:
\[
\begin{bmatrix}
\hat{Q}_m \\
\sqrt{P_m}(S\Phi_{sk})^T \\
* \\
\sqrt{P_m}(S\Phi_{sk})^T \\
\hat{S} + S^T - Q_0 \\
* \\
\hat{S} + S^T - Q_0 \\
* \\
\cdots \\
* \\
\hat{S} + S^T - Q_d
\end{bmatrix} > 0, 
\]
(15)
where \( \hat{Q}_m, \hat{S} \) are defined by (12). Inequality (15) can be rewritten as:
\[
\begin{bmatrix}
\hat{Q}_m \\
\sqrt{P_m}W^T \\
\hat{S} + S^T - Q_0 \\
\hat{S} + S^T - Q_0 \\
* \\
\hat{S} + S^T - Q_0 \\
* \\
\cdots \\
* \\
\hat{S} + S^T - Q_d
\end{bmatrix} > 0, 
\]
(16)
Then LMI (16) can be rewritten as:
\[
-\Omega_1 + \hat{M}\hat{\Gamma}^T(k)\hat{N} + \hat{N}^T\hat{\Gamma}^T(k)\hat{M} + \hat{L}\hat{\Gamma}_1(k)\hat{V} + \hat{V}^T\hat{\Gamma}_1(k)^T\hat{L}^T < 0,
\]
where \( \hat{\Gamma}^T(k) = \text{diag}[\hat{\Gamma}_1(k)] \), \( \hat{\Gamma}_1(k) = \text{diag}[\hat{\Gamma}_1(k)] \), \( \hat{\Gamma}_1(k) = \text{diag}[\hat{\Gamma}_1(k)] \), \( \hat{\Gamma}_1(k) = \text{diag}[\hat{\Gamma}_1(k)] \), \( \hat{\Gamma}_1(k) = \text{diag}[\hat{\Gamma}_1(k)] \).
From lemma 3, we know LMI (16) is satisfied if there exist positive scalars \( \epsilon, \epsilon \) satisfy:
\[
-\Omega_1 + \epsilon\hat{M}\hat{\Gamma}^T + \epsilon^{-1}\hat{N}^T\hat{\Gamma} + \epsilon\hat{L}\hat{\Gamma}_1(k)^T\hat{L}^T + \epsilon^{-1}\hat{\Gamma}^T\hat{V} < 0,
\]
(17)
By Schur complements, we know inequality (17) is equivalent to (16), then NCS (7) can be robust MS stabilized with the state feedback gain
\[
F = K^{-1}Y. 
\]
\[\square\]

4 Simulation Examples

To illustrate the efficiency of the proposed method, we give the following example.

Example 1. Consider the following NCS:
\[
x(k + 1) = \begin{bmatrix}
-0.1 & 3 \\
0 & -1.01
\end{bmatrix} x(k) + \begin{bmatrix}
0.001 \\
-0.002
\end{bmatrix} + \begin{bmatrix}
0.002 \\
0.003
\end{bmatrix} u(k).
\]
(18)
\[
u(k) = (F + [0.02 \ 0.03] \Gamma_1(k) \bar{\epsilon}(k)).
\]
(19)
The probability of packet transmission is expressed by a Markov transition probability matrix \( P \) with
\[
P = \begin{bmatrix}
0.48 & 0.51 & 0.01 \\
0.499 & 0.499 & 0.002 \\
0.48 & 0.51 & 0.01
\end{bmatrix}.
\]
Case (1): let \( \Gamma(k) = 0, \Gamma_1(k) = 0 \) which means there is no uncertainty in the system.
Solving the LMI in lemma 2 with LMI toolbox [1], we get
\[
S = \begin{bmatrix}
0.2116 & 0.0481 \\
0.0481 & 0.6983
\end{bmatrix}, \quad Q_0 = \begin{bmatrix}
-0.5836 & 0.0797 \\
0.0797 & 0.3651
\end{bmatrix},
\]
\[
Q_1 = \begin{bmatrix}
-0.5904 & 0.0787 \\
0.0787 & 0.3567
\end{bmatrix}, \quad Q_2 = \begin{bmatrix}
0.2197 & 0.0473 \\
0.0473 & 0.8868
\end{bmatrix},
\]
and the feedback gain \( F = [0 \ 0.8127] \). With the initial condition \( x(0) = [10 \ -10]^T \), the state values of the NCS with multiple packet transmission is shown in Fig.2.

![Fig. 2: The state values of the NCS in case (1)](image)

Case (2): let \( \Gamma(k) = \sin(20k), \Gamma_1(k) = \sin(50k) \) which means there is a certain degree of uncertainty in system.
Solving the LMI in Theorem 1 with LMI toolbox [1], we obtain
\[
\epsilon = 108.2585, \quad \epsilon = 108.2515, \quad Q_1 = \begin{bmatrix}
-98.7895 & 2.8417 \\
2.8417 & 29.3105
\end{bmatrix}, \quad Q_2 = \begin{bmatrix}
31.1067 & 4.7738 \\
4.7738 & 139.6529
\end{bmatrix},
\]
and the mode-independent controller \( F = [0.0014 \ 0.9247] \). With the initial condition \( x(0) = [10 \ -10]^T \), the state values of the NCS with multiple packets transmission is shown in Fig.3.

It can be seen from the figures that the NCS is robust MS stable. This example illustrates that the jump system approach proposed in this paper leads to effective results. It only requires plant state measurements be transmitted sparsely and the packet transmission sequence in the NCS is statistic.

5 Conclusion

In this paper, we dealt with stability and stabilization of uncertain NCS with multiple packet transmitted over a
shared channel. It was modeled as a class of discrete-time jump system. The packets transition sequence of the systems was governed by a finite state Markov chain. Sufficient condition on robust stability of the NCS was derived and a state feedback controller design method was also obtained. Finally, we gave an numerical example to illustrate the feasibility and effectiveness of our approach. The results suggested that data packet could be transmitted sparsely to save network bandwidth while preserving the stability of the NCS. The packet transmission sequence in the NCS was non-deterministic. This was of practical interest in the application of NCSs.

References


